Mathematical Studies

for the IB DIPLOMA

Second Edition

for 2012 syllabus

ANSWERS

Ric Pimentel
Terry Wall

FREE ONLINE RESOURCES

HODDER EDUCATION

Answers to revision exercises

1.2 Approximation

1 **a** **i** 3000 **ii** 2800 **iii** 2840
b **i** 13 000 **ii** 12 900 **iii** 12 940
c **i** 10 000 **ii** 9600 **iii** 9580
d **i** 0 **ii** 500 **iii** 500

2 **a** **i** 2.2 **ii** 2.18 **iii** 2.183
b **i** 0.9 **ii** 0.92 **iii** 0.918
c **i** 10.0 **ii** 9.96 **iii** 9.963
d **i** 0.0 **ii** 0.04 **iii** 0.039

3 **a** **i** 4 **ii** 3.9 **iii** 3.95
b **i** 20 **ii** 20 **iii** 20.4
c **i** 0.02 **ii** 0.015 **iii** 0.0155
d **i** 1 **ii** 0.98 **iii** 0.975

4 Student's answers may differ from the approximations given below.
a ≈ 3000 **b** ≈ 400 **c** ≈ 9
d ≈ 20 **e** ≈ 20 **f** ≈ 10000

5 Student's answers may differ from the approximations given below.
a $\approx 60\,\text{cm}^2$ **b** $\approx 100\,\text{cm}^2$ **c** $\approx 580\,\text{cm}^2$
d $\approx 100\,\text{cm}^2$

6 **a** $60.27\,\text{cm}^2$ **b** $98.04\,\text{cm}^2$ **c** $608.16\,\text{cm}^2$
d $93.8\,\text{cm}^2$
b Student's percentage error calculations

7 **a** 19.055 kg **b** 17.945 kg

8 **a** $201.061\,93\,\text{cm}^2$
b Area = $192\,\text{cm}^2$, percentage error = −4.5% (2 s.f.)
c Area = $201.142\,86\,\text{cm}^2$, percentage error = 0.040% (2 s.f.)

9 **a** 64.4°F **b** 66°F **c** 2.5%
d 4.7% **e** 10°C

10 **a** $58.86\,\text{m s}^{-1}$ **b** $60\,\text{m s}^{-1}$
c 1.9% (2 s.f.)

1.3 Standard form

1 b and d

2 **a** 3.2×10^4 **b** 6.2×10^2 **c** 7.77×10^8
d 9×10^4 **e** 8×10^6 **f** 4.85×10^7

3

London to Tokyo	9.6×10^3 km
London to Paris	3.4×10^2 km
London to Wellington	1.9×10^4 km
London to Cambridge	7.8×10^1 km
London to Cairo	3.5×10^3 km

4 **a** 3×10^6 **b** 9×10^6 **c** 7.8×10^7
d 4×10^{13}

5 **a** 4×10^{-2} **b** 7.6×10^{-3} **c** 5×10^{-6}
d 3.04×10^{-2}

6 8.7×10^{-4} 3.6×10^{-3} 1.4×10^{-2}
2.5×10^{-2} 7.4×10^{-2} 9.8×10^{-1}

7 **a** 7.5×10^{-4} **b** 5×10^{-4}
c 1.67×10^{-4} **d** 1.67×10^{-9}

8 **a** −4 **b** −5 **c** −5
d 3

9 2×10^4 seconds

10 4.00×10^7 m

1.4 SI units of measurement

1 **a** Student's estimate in kilograms.
b Student's estimate in metres.
c Student's estimate in metres.
d Student's estimate in litres.
e Student's estimate in kilometres.
f Student's estimate in grams.

2 **a** 200 mm **b** 35 000 m
c 4.6 cm **d** 0.06 km
e 320 000 mm **f** 0.000 095 km

3 **a** 0.1 t **b** 0.06 kg
c 3600 kg **d** 14 000 mg
e 8 670 000 mg **f** 0.002 560 t

4 **a** 2.6 litres **b** 0.08 litres
c 1650 ml **d** 85 ml

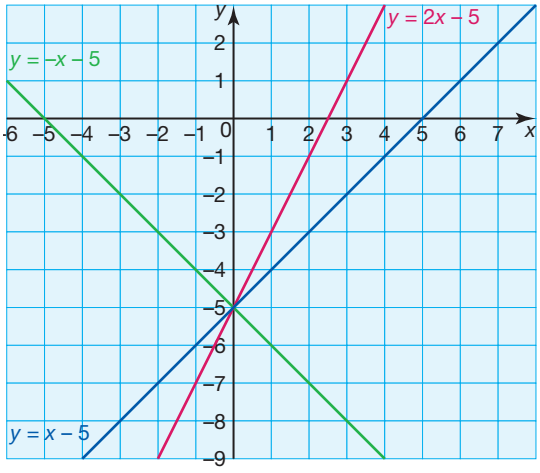
5 375.92 kg

6 2.013 m

7 1.062 litres

1.6 Graphical solution of equations

1

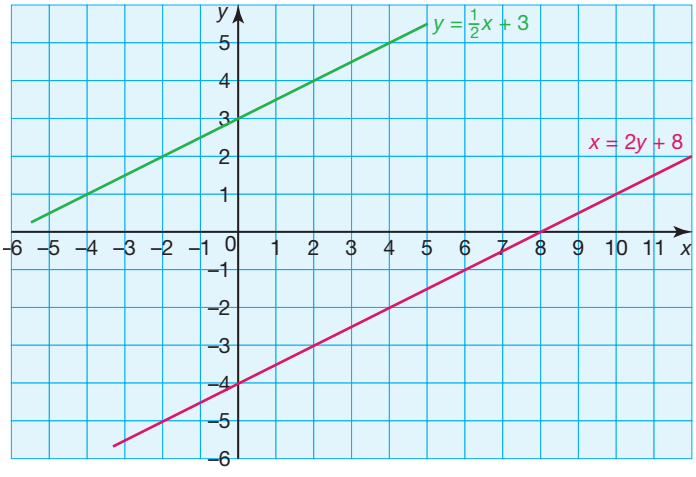


The graphs intercept at (0, –5).

2 Possible answers: $y = 2x + 2$, $y = 2x$, $y = -x + 2$

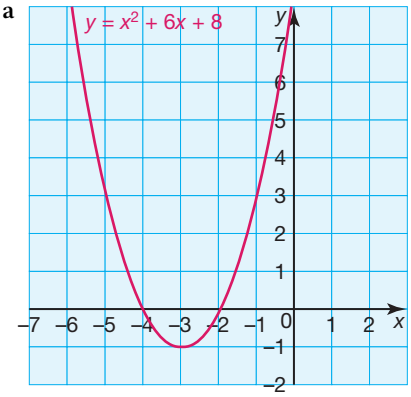
3 **a** (3, 5) **b** (–2, 4) **c** (–3.5, 17) **d** (6, 0)

4 **a**

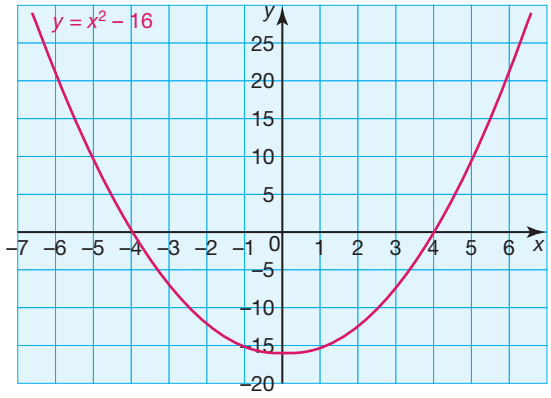


b The lines are parallel.

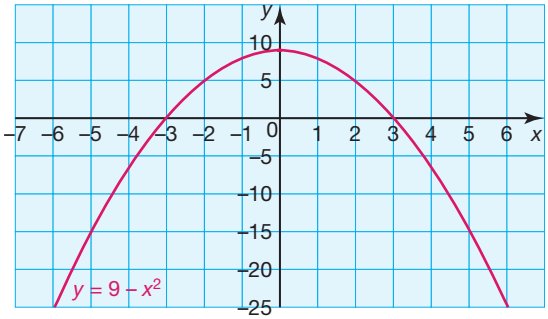
5 a



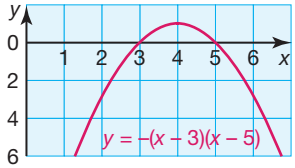
b



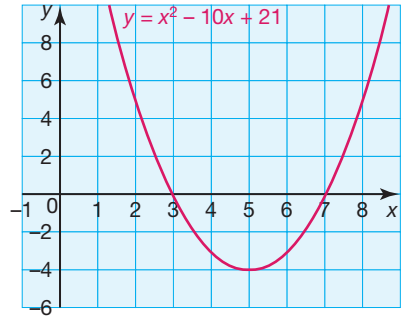
c



d

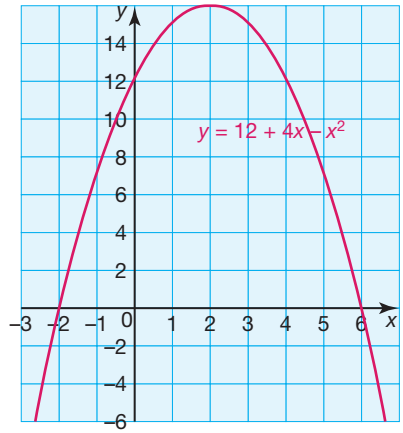


6 a i



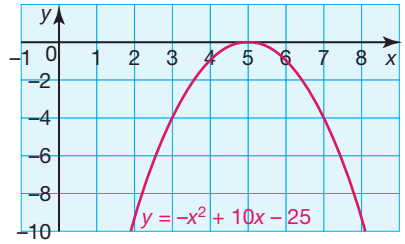
ii (3, 0) and (7, 0)

b i



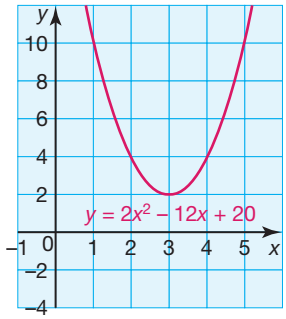
ii (–2, 0) and (6, 0)

c i



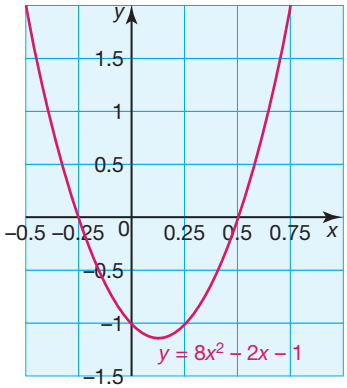
ii (5, 0)

d i



ii No real roots

e i



ii $\left(-\frac{1}{4}, 0\right)$ and $\left(\frac{1}{2}, 0\right)$

1.7 Arithmetic sequences and series

1 a i $u_2 = 9, u_3 = 3, u_4 = -3$
ii Arithmetic
b i $u_2 = -3, u_3 = 15, u_4 = 3$
ii Not arithmetic
c i $u_2 = 3.5, u_3 = 12.5, u_4 = 39.5$
ii Not arithmetic
d i $u_2 = 2.5, u_3 = 0, u_4 = -2.5$
ii Arithmetic

2 a i $5n - 1$ **ii** 99
b i $-8n + 11$ **ii** –149
c i $2.5n - 7$ **ii** 43
d i $-0.25n + 3.75$ **ii** –1.25

3 a i 6 **ii** $6n - 31$
b i –3.5 **ii** $-3.5n + 10.5$
c i 7 **ii** $7n - 40$
d i –4.2 **ii** $-4.2n + 48.4$

4 a $1 + 3 + 5 + 7 + 9$
b $3 + 2 + 1 + 0 + -1$
c $5 + 4.5 + 4 + 3.5 + 3 + 2.5 + 2$
d $3 + 0 + -3 + -6$

5 a $\sum_{1}^{5} 4n - 2$ **b** $\sum_{1}^{6} -2n + 7$
c $\sum_{1}^{6} \frac{3}{2}n - 2$ **d** $\sum_{1}^{4} -0.1n - 4$

6 a 3825 **b** 175
c 286 **d** 406

7 a 2 **b** –18
c 20 **d** 20

8 a 25 **b** 4
c 1260

9 a $0.3x$ **b** 7
c 205

10 25

1.8 Geometric sequences and series

1 Successive terms of an arithmetic sequence have a common difference, whilst successive terms of a geometric sequence have a common ratio.

2 a i $u_2 = 2, u_3 = 10, u_4 = 42$
ii Not geometric
b i $u_2 = -3, u_3 = 9, u_4 = -27$
ii Geometric
c i $u_2 = 15, u_3 = 37.5, u_4 = 93.75$
ii Geometric
d i $u_2 = -1, u_3 = 4, u_4 = -1$
ii Not geometric

3 a i 3 **ii** 405, 1215
iii $u_n = 5(3)^{n-1}$
b i $\frac{1}{6}$ **ii** $1, \frac{1}{6}$
iii $u_n = 1296\left(\frac{1}{6}\right)^{n-1}$
c i $\frac{2}{3}$ **ii** $7\frac{1}{9}, 4\frac{20}{27}$
iii $u_n = 36\left(\frac{2}{3}\right)^{n-1}$
d i $-\frac{5}{2}$ **ii** $156\frac{1}{4}, -390\frac{5}{8}$
iii $u_n = 4\left(-\frac{5}{2}\right)^{n-1}$

4 **a** $u_1 = -3, u_2 = -12, u_3 = -48$
b 7

5 **a** $\frac{1}{3}$ **b** 81 **c** $\frac{1}{243}$

6 \$71 647

7 **a** 363 **b** –728 **c** 699 040
d $\frac{182}{243}$

8 **a** $\frac{2}{5}$ **b** $\frac{125}{2}$ **c** 103.996

9 **a** 10 **b** $\frac{5}{4}, \frac{5}{2}$ **c** $\frac{5115}{4}$

10 9

1.9 Simple interest and compound interest

1 **a** £82.50 **b** \$4800 **c** €2187.50

2 3.5%

3 5 years

4 **a** Simple interest, as the amount added is constant. i.e. there is a common difference.
b 2.5%
c $T = 15\,000 + 375n$
d $I = 375n$

5 **a** Compound interest, as there is a common ratio between successive terms.
b 10% per year
c $T = 15000 \times 1.1^n$
d $I = 15000 \times 1.1^n - 15000$

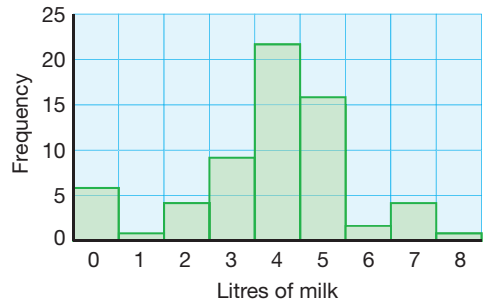
6 **a** £24.36 **b** \$2969.24 **c** €9953.45

7 **a** **i** €10 625 **ii** €9031.25 **b** 16 years

8 **a** €764.06 **b** €776.21 **c** €782.47
d €786.72

2.3 and 2.5 Grouped discrete or continuous data and Measures of central tendency

1 **a**



b 4 **c** 3.9 **d** 4

2 **a** $15 \leq M < 20$
b 17.7 kg
c $15 \leq M < 20$; the 153rd case falls in this group.

3 **a** 76 kg **b** 85 kg

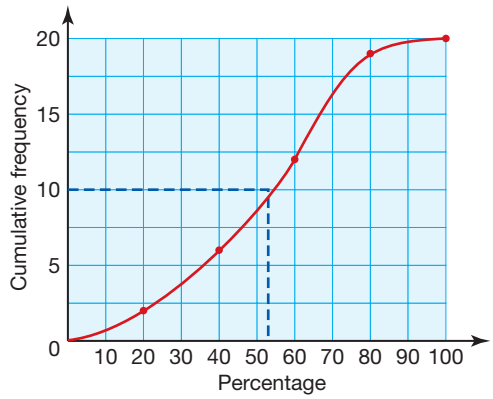
4 **a** 19 **b** 1.03 euros

2.4 Cumulative frequency

1 **a**

Percentage	Frequency	Cumulative frequency
$0 \leq x < 20$	2	2
$20 \leq x < 40$	4	6
$40 \leq x < 60$	6	12
$60 \leq x < 80$	7	19
$80 \leq x < 100$	1	20

b

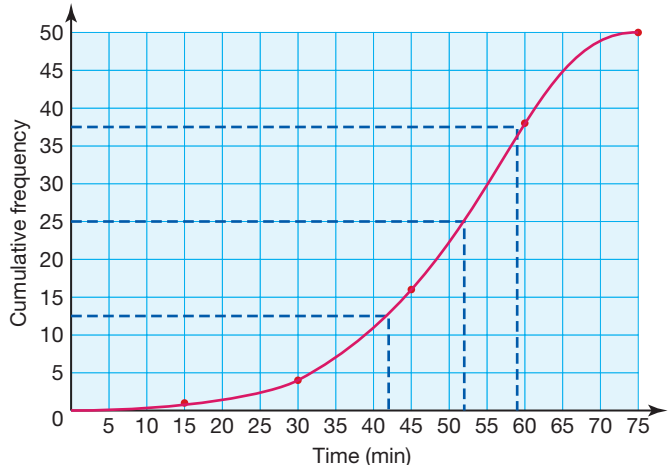


c Median ≈ 53%

2 a

Time (min)	Frequency	Cumulative frequency
$0 \leq t < 15$	1	1
$15 \leq t < 30$	3	4
$30 \leq t < 45$	12	16
$45 \leq t < 60$	22	38
$60 \leq t < 75$	12	50

b



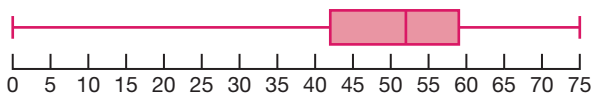
c Median ≈ 52 minutes

d Lower quartile ≈ 42 minutes

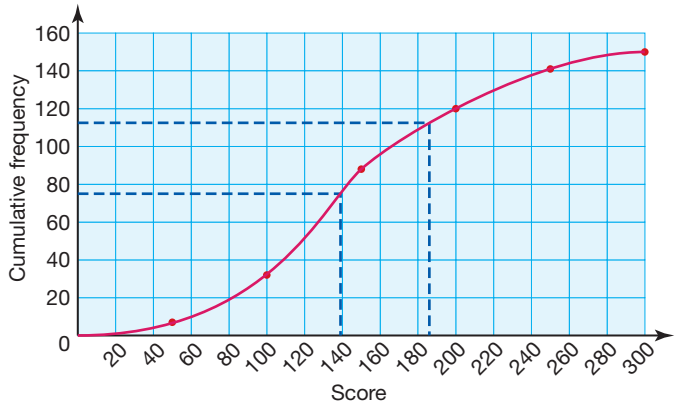
Upper quartile ≈ 59 minutes

Middle 50% take between 42 and 59 minutes.

e



3 a



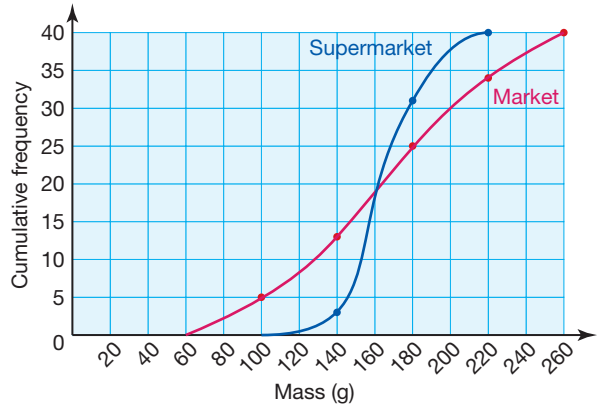
b Median ≈ 138

c Top 25% represents the upper quartile.

Upper quartile ≈ 187

Any score ≥ 187 will be in the top 25%.

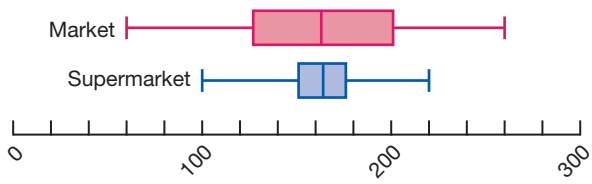
4 a



b

	Minimum value	Lower quartile	Median	Upper quartile	Maximum value
Market	60	127	163	201	260
Supermarket	100	151	164	178	220

c



2.6 Measures of dispersion

1 a i 20 **ii** 9
b i 26 **ii** 17.5

2 a 5 **b** 3

3 a 1.81 **b** 8.65

4 a Mean A = 24.1 °C
Mean B = 22.9 °C
b Range A = 5 °C
Range B = 18 °C
c IQR A = 2 °C
IQR B = 12 °C
d Standard deviation A = 1.41 °C
Standard deviation B = 6.26 °C
e The temperature at resort A is more consistent as it has a lower standard deviation.

5 a Mean A = 11.0 s
Mean B = 10.5 s
b Standard deviation A = 0.12s
Standard deviation B = 0.59s
c Sprinter B is faster as he has the lower mean.
d Sprinter A is more consistent as his standard deviation is lower than that of B.

6 Class B is likely to have students of similar ability as the standard deviation is lower than that for Class A. This means that the results are less spread out.

3.1 and 3.2 Logic and Sets and logical reasoning

1 a Proposition; true
b Proposition; false
c Not a proposition (What is y?)
d Proposition; true
e Proposition; indeterminate (no location is given)
f Not a proposition (Questions are not propositions.)

2 a $p \wedge q$ **b** $p \vee q$ **c** $p \veebar q$

3.3 and 3.4 Truth tables and Implication; converse; inverse; contrapositive and logical equivalence

1

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

1	2	3
H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

2 $p \wedge q$ states that both occur; $p \veebar q$ states that both cannot occur at the same time, therefore there is a contradiction.

3

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \veebar q$
T	T	F	T	T	F
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	F	F	F

4

p	q	$\neg p$	$\neg q$	$(\neg p) \wedge (\neg q)$	$p \vee q$	$\neg(p \vee q)$
T	T	F	F	F	T	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	T	F	T

5 a i If a number is odd then it is divisible by two. (False)
ii If a number is divisible by two then it is odd. (False)
If a number is not odd then it is not divisible by two. (False)

If a number is not divisible by two then it is not odd. (False)

b i If a shape has eight sides then it is an octagon. (True)

ii If a shape is an octagon then it has eight sides. (True)
If a shape does not have eight sides then it is not an octagon. (True)
If a shape is not an octagon then it does not have eight sides. (True)

c i If a solid is an icosahedron then it has twelve faces. (False)

ii If a solid has twelve faces then it is an icosahedron. (False)
If a solid is not an icosahedron then it does not have twelve faces. (True) (*Note*: logically true, but not necessarily true in reality as the original proposition is false.)
If a solid does not have twelve faces then it is not an icosahedron. (True) (*Note*: logically true)

d i If two triangles are congruent then they are similar. (True)

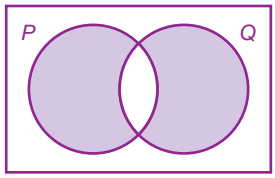
ii If two triangles are similar then they are congruent. (False)
If two triangles are not congruent then they are not similar. (False)
If two triangles are not similar then they are not congruent. (True)

6 a

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$(p \Rightarrow q) \veebar (q \Rightarrow p)$
T	T	T	T	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	F

b If p and q are either both true or both false, then $(p \Rightarrow q) \veebar (q \Rightarrow p)$ is false, i.e. p and q cannot happen together.

7 a



b $(P \cap Q') \cup (Q \cap P')$

3.5 Set theory

1 a i Countries in Africa

ii Two other African countries

b i Multiples of 3

ii Two other multiples of 3

c i Rivers

ii Two other rivers

d i Fraction where numerator = denominator – 1

ii Two other fractions where the numerator = denominator – 1

2 a $B = \{22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48\}$

b $C = \{23, 29, 31, 37, 41, 43, 47\}$

c $D = \{25, 36, 49\}$

3 $Q = \{a, b, c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a\}, \{b\}, \{c\}$

4 a True **b** False **c** False **d** True

5 $A' = \{5, 10, 15, 20, 25, 30\}$

6 $P' = \{\text{Tuesday, Wednesday, Thursday, Friday, Saturday}\}$

7 a $M \cup N = \{\text{Alex, Johanna, Sarah, Vicky, Asif, Gabriella, Pedro, Frances, Raul, Luisa}\}$.

b $M \cap N = \{\text{Alex, Gabriella}\}$

c $M \cap N' = \{\text{Johanna, Sarah, Vicky, Asif, Pedro}\}$

8 a $E \cup F = \{\text{positive integers}\}$

b $E \cap F = \varnothing$ – the empty set

c $E' \cap F = \{\text{odd numbers}\}$

3.6a Sample space

1 $S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

2 $S = \{RR, RY, RG, YY, YG, YR, GG, GR, GY\}$

3 a Pass and fail

b $S = \{PP, PF, FP, FF\}$

4 a Win and lose

b $S = \{5{-}0, 4{-}1, 3{-}2, 2{-}3, 1{-}4, 0{-}5\}$

5 a 8 events

b $S = \{BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG\}$

6 a Red and black
b S = {RRR, RRB, RBR, BRR, RBB, BRB, BBR, BBB}

3.6b and 3.7a Probability and Combined events

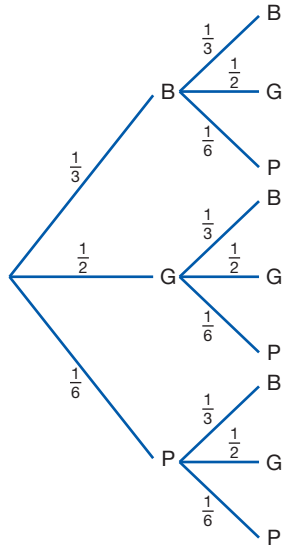
1 a i $\frac{10}{24} = \frac{5}{12}$ **ii** $\frac{16}{24} = \frac{2}{3}$
b i $\frac{10}{23}$ **ii** $\frac{13}{23}$

2 a

		Six-sided dice					
		1	**2**	**3**	**4**	**5**	**6**
Four-sided dice	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10

b $\frac{3}{24} = \frac{1}{8}$ **c** $\frac{4}{24} = \frac{1}{6}$
d 6

3 a b



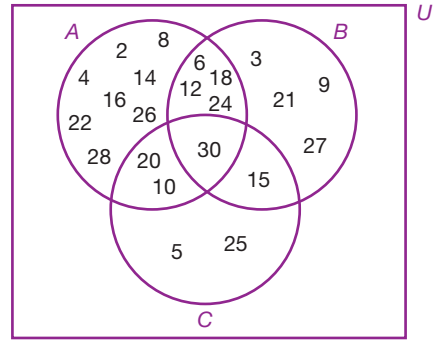
c $\frac{1}{9}$ **d** $\frac{5}{9}$

4 a $\frac{1}{12}$ **b** $\frac{8}{27}$
c $1 - \frac{1}{1728} = \frac{1727}{1728}$

5 a $\frac{7}{2990}$ **b** $\frac{146}{1495}$

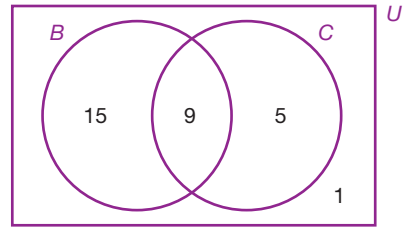
6 a 40 **b** $\frac{17}{40}$ **c** $\frac{3}{20}$
d $\frac{1}{20}$ **e** $\frac{31}{40}$ **f** $\frac{19}{40}$

7 a



b i $\frac{15}{22}$ **ii** $\frac{14}{22}$ **iii** $\frac{5}{22}$

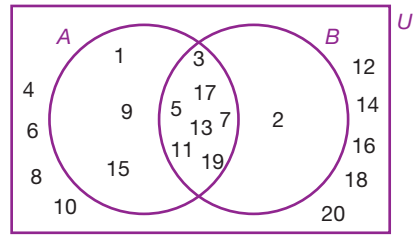
8 a



b i $\frac{1}{5}$ **ii** $\frac{29}{30}$ **iii** $\frac{1}{2}$

3.7b Venn diagrams

1



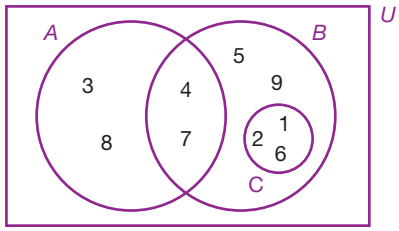
2 a $P = \{3, 6, 9, 12, 15, 18\}$
b $R = \{6, 12, 18\}$
c $P \cap Q = \{6, 12, 18\}$
d $P \cup Q = \{2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20\}$
e $P' \cap Q = \{2, 4, 8, 10, 14, 16, 20\}$

3 a $L \cap N = \{7, 9, 16, 18\}$
b $N \cup M = \{2, 3, 5, 6, 7, 9, 11, 12, 14, 15, 16, 18, 20\}$

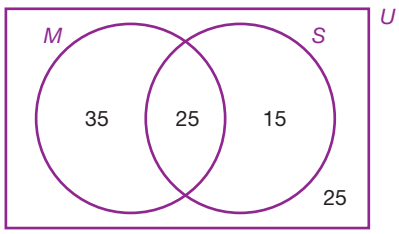
c $L \cap M \cap N = \{9, 18\}$
d $N' \cap L = \{1, 5, 10, 14, 19\}$
e $N' \cup L' = \{2, 4, 8, 11, 13, 17, 20\}$
f $M' \cup L \cap N = \{3, 7, 12, 16\}$

4 **a** **i** True **ii** False **iii** True
b **i** 8 **ii** 6 **iii** 3
iv 7 **v** 1 **vi** 1

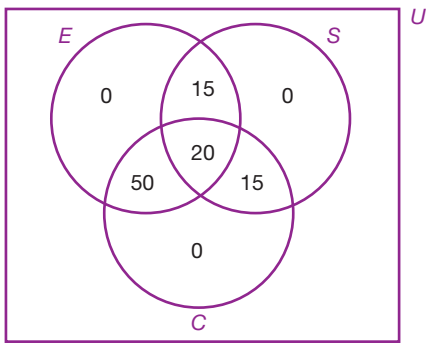
5



6



7



3.7c Laws of probability

1 **a** 0.93 **b** 0.53

2 **a** $\frac{1}{4}$ **b** $\frac{3}{4}$ **c** $\frac{1}{2}$

3 $\frac{30}{49}$

4 **a** 0.2 **b** 0.8 **c** 0.45
d 0.69

5 **a** 0.68 **b** 0.88

6 0.95

4.1 The normal distribution

1 **a** Approximately 68%
b Approximately 95%

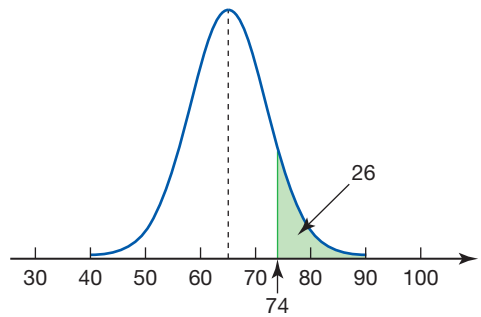
2 **a** Mean, median and mode the same as the distribution peaks coincide.
b Standard deviations are different. Standard deviation for Q is greater as the distribution is more spread out.

3 **a** More likely to belong to Y as 50% of species Y are greater than 150 cm in length.
b 0.0062

4 **a** 4.78% **b** 4 bags

5 **a** 78.7 **b** $57.74 \leq C \leq 62.26$

6 **a**



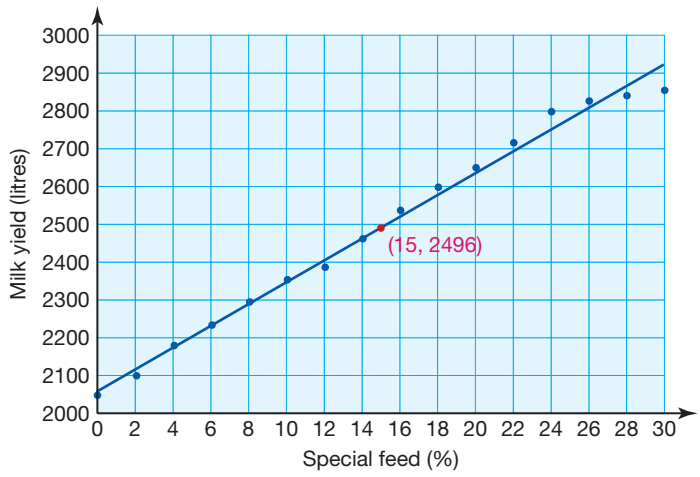
b 262

4.2 and 4.3 Scatter diagrams, bivariate data and linear correlation and The regression line for *y* on *x*

1 The following are only likely correlations. Student's answers may vary if supported by clear justifications.
a Moderate positive correlation
b Moderate/strong positive correlation
c Weak/no correlation
d Strong negative correlation
e Strong negative correlat ion
f Strong positive correlation
g No correlation
h Weak negative/no correlation

2 **a** Strong positive correlation
b Weak negative correlation

3 a–d



e Approximately 2496 litres
f $y \approx 29x + 2060$
g Approximately 4960 litres
h Unlikely as it assumes that the relationship continues to be linear. The graph appears to tail off after 24% of special feed.

4 a $r = 0.945$
b r is close to +1, indicating a strong positive correlation between number of people in the store and money taken at the tills.

5 a $y = -0.2x + 991$
b Rate of fuel consumption (number of litres used per kilometre travelled)
c 391 km
d 4955 km

4.4 The χ^2 test for independence

1 a 2
b

	Distinction	Pass	Fail	Total
Male	16.5	93.6	19.9	130
Female	21.5	122.4	26.1	170
Total	38	216	46	300

c i Failing is independent of gender.
ii Failing is dependent on gender.
d $\chi^2 = 7.75$
e $7.75 > 5.991$ therefore reject H_0. i.e. Failing is dependent on gender.

2 a 3

b

	Romance	Horror	Action	Comedy	Total
Male	10.8	11.2	9.6	18.5	50
Female	17.2	17.8	15.4	29.5	80
Total	28	29	25	48	130

c i Viewing preferences are independent of gender.
ii Viewing preferences are dependent on gender.
d $\chi^2 = 7.75$
e Viewing preferences are dependent on gender at all three levels of significance.
i $15.21 > 6.251$
ii $15.21 > 7.815$
iii $15.21 > 9.348$

3 a Level of satisfaction is independent of age.
b $\chi^2 = 18.13$
c i $18.13 < 21.66$
Null hypothesis accepted at 1% significance.
ii $18.13 > 16.919$
Null hypothesis rejected at 5% significance.
iii $18.13 > 14.684$
Null hypothesis rejected at 10% significance.

5.1 Straight lines

1 a 4 **b** -2.5

2 a $-\frac{1}{4}$ **b** $\frac{1}{6}$ **c** $-\frac{3}{2}$
d $\frac{4}{7}$

3 a 13 **b** 10

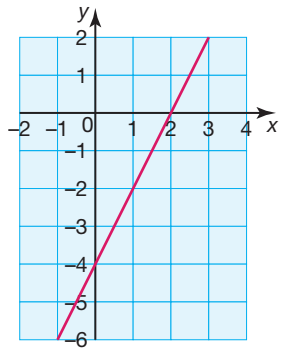
4 a $(7, 6.5)$ **b** $(-1, 11)$

5 a $y = 3x - 1$ **b** $y = \frac{1}{2}x + 1$

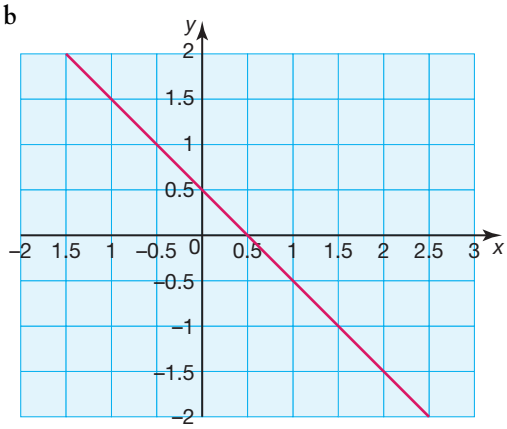
6 a gradient = 1 y-intercept = -2
b gradient = -3 y-intercept = 1
c gradient = 2 y-intercept = -3
d gradient = $-\frac{5}{3}$ y-intercept = 4

7 a i $y = 4x - 1$ **ii** $4x - y - 1 = 0$
b i $y = -2x + 3$ **ii** $2x + y - 3 = 0$

8 a



b

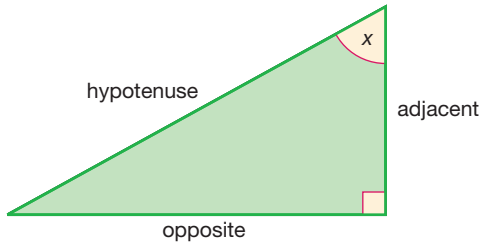


9 a $x = 1$ $y = 3$ **b** $x = 2$ $y = 1$

10 a $3t + c = 4.40$, $2t + 3c = 4.80$
b i $t = \$1.20$ **ii** $c = \$0.80$

5.2 Right-angled trigonometry

1



2 a 24.8° **b** 57.0°

3 a 5.2 cm **b** 8.8 cm

4 a $x = 43.0°$ $a = 8.8$ cm
b $a = 9.1$ cm $x = 28.9°$

5 a 30.2 km **b** 124.2°

6 a 6.7° **b** 120.8 m

7 52.3 km

5.3 Trigonometry and non-right-angled triangles

1 a $x = 40.5°, 139.5°$ **b** $x = 14.5°, 165.5°$
c $x = 90°$ **d** $x = 0°, 180°$

2 a $\theta = 60°, 300°$ **b** $\theta = 113.6°, 246.4°$
c $\theta = 180°$

3 8.4 cm

4 44.0°

5 a 7.7 cm **b** 46.5°

6 75.6 cm^2

7 5.8°

8 a Student's construction
b 41.8°

5.4 and 5.5 Geometry of three-dimensional shapes

1 a 14.1 cm **b** 17.3 cm **c** 35.3°

2 a 5.7 cm **b** 13.9 cm **c** 4 cm
d 73.9°

3 a 174 cm^2 **b** 339.3 cm^2

4 a 6.2 cm **b** 242.2 cm^2 **c** 171.7 cm^3
d 27.3°

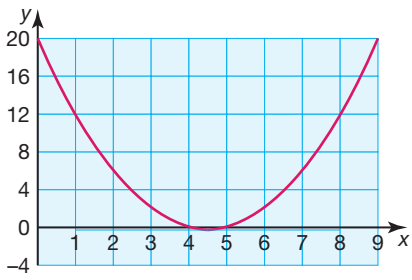
5 a 63.5 cm **b** 444.7 cm^2

6 4.3 cm

7 a 75.4 cm **b** 41.0 cm

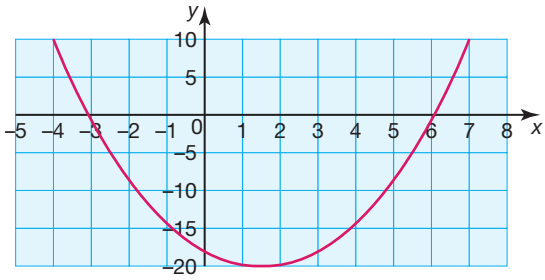
6.3 Quadratic functions and their graphs

1 a i



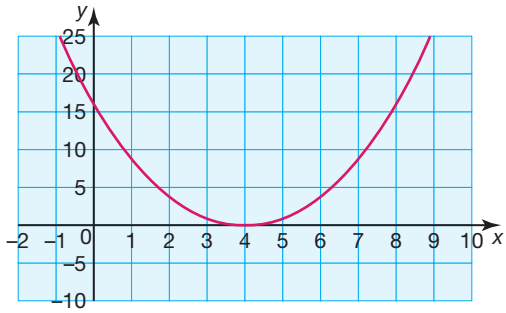
ii $x = 4$ and 5 **iii** $y = 20$

b i



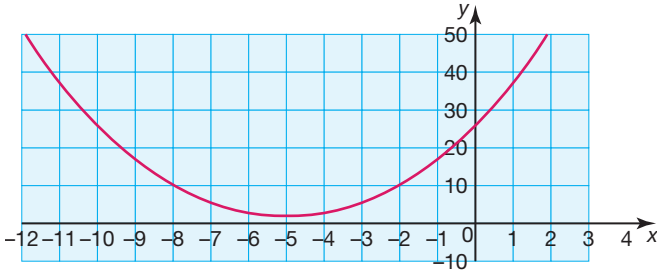
ii $x = -3$ and 6 **iii** $y = -18$

c i



ii $x = 4$ **iii** $y = 16$

d i



ii No x-intercept **iii** $y = 27$

2 **a** $x = 1$ **b** $x = -2$
c $x = 2.5$ **d** $x = 1.5$

3 **a** Student's equation
b Student's equation

4 **a** $f(x) = (x + 5)(x + 6)$
b $f(x) = (x - 2)(x + 6)$
c $f(x) = (-x + 3)(x - 5)$
d $f(x) = (x - 6)(x + 6)$

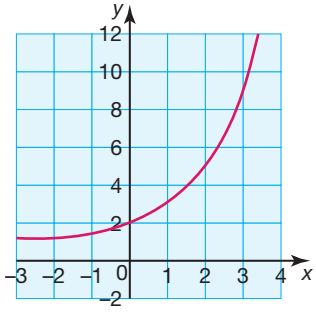
5 **a** $(x + 1)(x - 4) = 0 \Rightarrow x = -1$ and 4
b $(x + 4)(x - 6) = 0 \Rightarrow x = -4$ and 6
c $(-x + 2)(x - 8) = 0 \Rightarrow x = 2$ and 8
d $(x - 4)(x - 7) = 0 \Rightarrow x = 4$ and 7

6 **a** $x = 3.1$ and -8.1 (1 d.p.)
b $x = 2.2$ and -11.2 (1 d.p.)
c $x = -0.5$ and -1.5
d $x = 6.8$ and 2.2 (1 d.p.)

7 **a** **i** $x(x + 6) = 16$ **ii** $x = 2$
b **i** $2x(x + 2) = 48$ **ii** $x = 4$

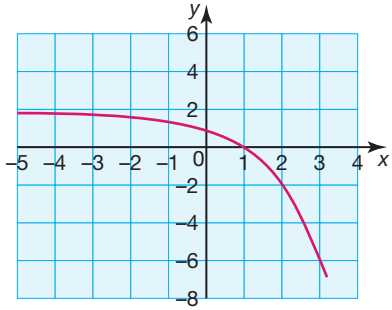
6.4 Exponential functions and their graphs

1 **a** **i**



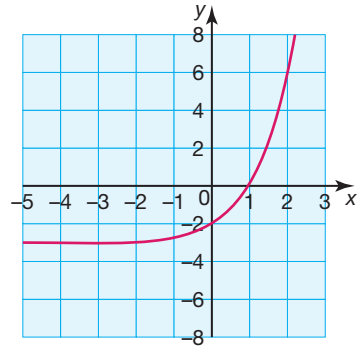
ii $y = 1$

b **i**



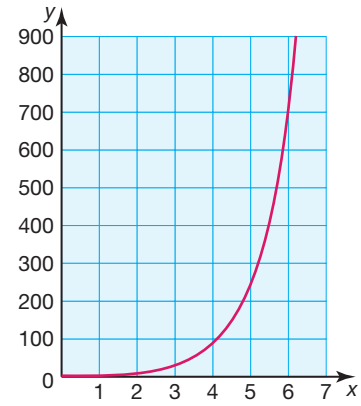
ii $y = 2$

c **i**



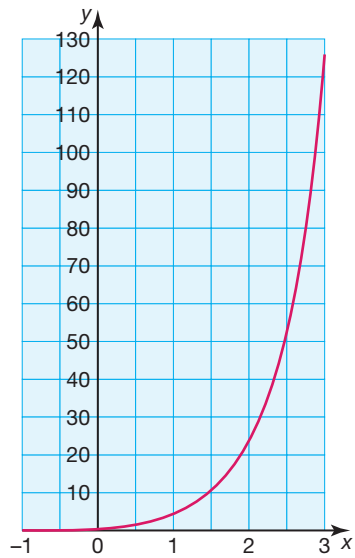
ii $y = -3$

2 **a** 6 cm **b** 86 cm **c** 5 h
d



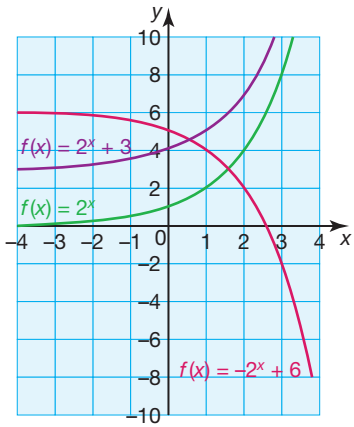
e $t \approx 4.1$ h

3 **a**



b **i** $x \approx 2.9$ **ii** $x \approx 2.4$

4



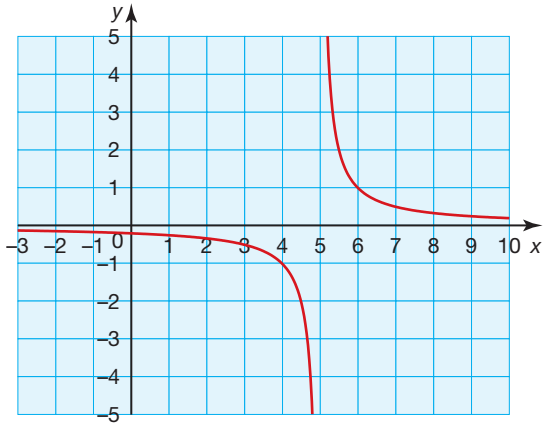
5 120 000 years

6.5, 6.6 and 6.7 Sketching and drawing graphs

1 a $x = 5$ and $y = 0$

b $\left(0, -\frac{1}{5}\right)$

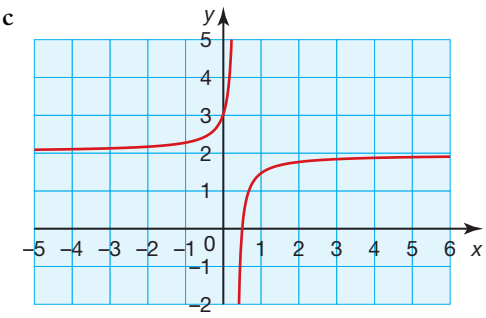
c



2 a $x = \frac{1}{3}$ and $y = 2$

b (0, 3) and $\left(\frac{1}{2}, 0\right)$

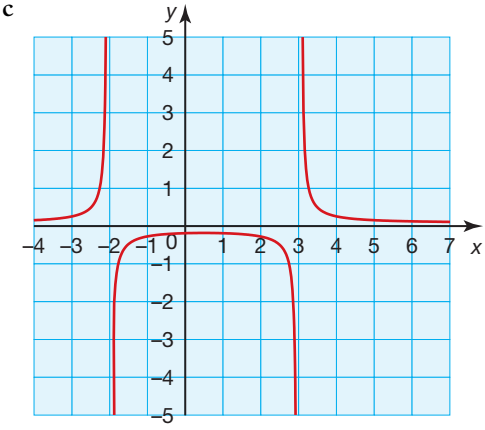
c



3 a $x = -2$ and 3, and $y = 0$

b $\left(0, -\frac{1}{6}\right)$

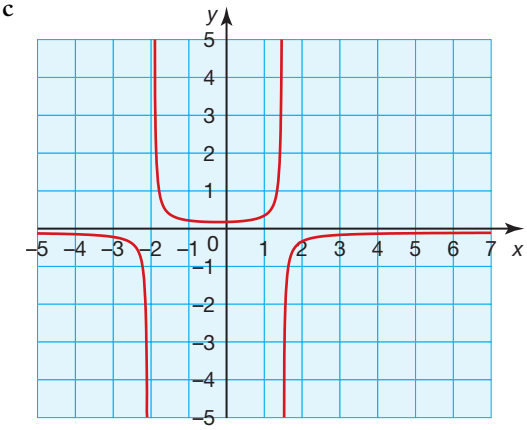
c



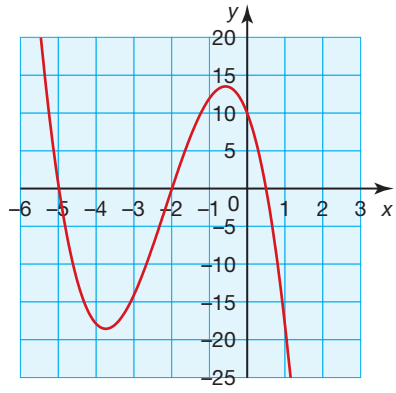
4 a $x = \frac{3}{2}$ and -2, and $y = 0$

b $\left(0, \frac{1}{6}\right)$

c



5 a



b $\left(\frac{1}{2}, 0\right)$, $(-2, 0)$, $(-5, 0)$ and $(0, 10)$

c $y = -(2x - 1)(x + 2)(x + 5)$

6 $x = 1, -4$ and $\frac{1}{2}$

7.2 and 7.3 Differentiation and The gradient of a curve at a given point

1 a $f'(x) = 2x + 3$
b $f'(x) = x - 5$
c $f'(x) = 6x^2 - 8x$
d $f'(x) = 2x^5 - 2x^3$

2 a $-x^{-2}$
b $-6x^{-4}$
c $-2x^{-3} - 2x^{-2}$
d $-6x^{-3}$

3 a $2x - 3$
b $6x^2 + 8x$
c $2x + 1$
d $3x^2 + 2x - 12$

4 a $4x - 1$

b $2x^2 - \frac{1}{3}$

c $-x^{-2} + 4x^{-3}$
d $4x - 12 - 6x^{-2}$

5 a $2x - 4$
b i 2 **ii** 0 **iii** −4

6 a $x - 4$
b i 4 **ii** 6 **iii** −1

7 a $\frac{dy}{dx} = 3x^2 + 4$

b $\frac{dy}{dx} \geq 4$ as a value less than 4 would give x as the square root of a negative number.

c i ± 1 **ii** 0 **iii** ± 3

8 a $f'(x) = 3x^2 - 13$
b 14
c 14
d $y = 14x - 42$
e $-\frac{1}{14}$
f $x + 14y - 3 = 0$

9 a $f'(x) = -2x - 2$
b Verify that both $(-2, 8)$ and $(1, 5)$ satisfy the equation $y = -x^2 - 2x + 8$.
c Gradient at A = 2
Gradient at B = −4
d $y = 2x + 12$
e $y = \frac{1}{4}x + \frac{19}{4}$
f By solving simultaneously, the point of intersection is at $\left(-\frac{29}{7}, \frac{26}{7}\right)$.

7.4, 7.5 and 7.6 Increasing and decreasing functions, Stationary points and Optimization

1 $x < -3$

2 a i $f'(x) = 2x$
ii $x > 0$
b i $f'(x) = 2x - 10$
ii $x > 5$
c i $f'(x) = -2x + 8$
ii $x < 4$
d i $f'(x) = 3x^2 - 4x - 8$
ii $x < -\frac{4}{3}, x > 2$

3 $f'(x) = 2x^2 + 1$. As $x^2 \geq 0$ for all values of x, $f'(x) \geq 1$ for all values of x.
Therefore $f(x)$ is an increasing function for all values of x.

4 $k \leq 1$

5 a 1
b Minimum

6 a $f'(x) = x^2 + x - 20$
b $x = 4$ and -5

c These are the x values where stationary points occur.

d As the coefficient of $x^3 > 0$, the graph takes a shape of the form

Therefore maximum occurs at $x = -5$ and minimum at $x = 4$

7 a $f'(x) = 3x^2 - 12$

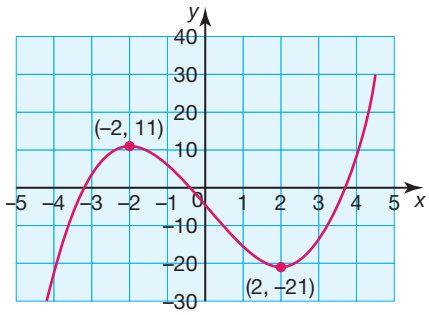
b $x = \pm 2$

c $(-2, 11)$ and $(2, -21)$

d Maximum at $(-2, 11)$ and minimum at $(2, -21)$

e $y = -5$

f



8 a $25 - x$ and $20 - 2x$

c 4.4 cm

d 1015 cm^3

e When $x = 4.3$ $\quad \frac{dV}{dx} = 8.94$

When $x = 4.5$ $\quad \frac{dV}{dx} = -8.50$

Gradient changes from positive to negative, therefore stationary point is a maximum